ENOB Calculation for ADCs with Input-Correlated Quantization Error Using a Sine-Wave Test

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***Abstract*—The equation for calculating ENOB from SNDR of a sine-wave test is only accurate when noise is uncorrelated to the input. In this paper, the equation for calculating ENOB from SNDR is derived for an ideal and a uniform stochastic ADC. The result of these derivations shows that calculating ENOB from SNDR using the conventional equation causes a better-than-actual result in the case a uniform stochastic ADC.**

I. Introduction

One of the important metrics for measuring the performance of an analog-to-digital converter (ADC) is the signal-to-noise ratio (SNR) which is frequently described in terms of effective number of bits (ENOB). The value of ENOB describes the overall accuracy of the converter. The most widely used method to determine SNR is to apply a sine-wave input and create a coherent FFT-plot. SNR is easily calculated by taking the power from the signal histogram bin and comparing it the the total remaining power from the remaining bins (after removing the DC bin). Creating an accurate, noise-free (through the use of band-pass filters) sine-wave input is very easy, making this method popular. The accepted conversion to ENOB from SNR from this sine-wave test is defined as

$$ENOB = \frac{SNR - 1.76}{6.02}, \tag{1}$$

where SNR has units of dB and ENOB is measured in bits. In this paper it will be demonstrated that while (1) holds true for ADCs with quantization noise that is mostly uncorrelated to the input, if quantization noise is correlated to the input a different conversion of SNDR to ENOB should be considered. This is important because conventional ADCs usually can claim uncorrelated quantization noise, but this is not always the case. In this paper, a uniform distribution stochastic flash ADC is considered as an example. Also, in this paper, only quantization noise is considered.

II. Ideal ADC

In this section SNR from quantization noise will be derived for a uniformly distributed input (ramp, triangle, etc.) and for a sine-wave input for an ideal ADC (Fig. 1(a)). Since we ultimately want to calculate ENOB with respect to a uniform input, it will then be shown that (1) converts sine-wave SNDR to uniform-input ENOB.

A. ENOB from Uniform Input

To calculate SNR, the signal power and noise power must be determined. First, consider the signal as being uniformly

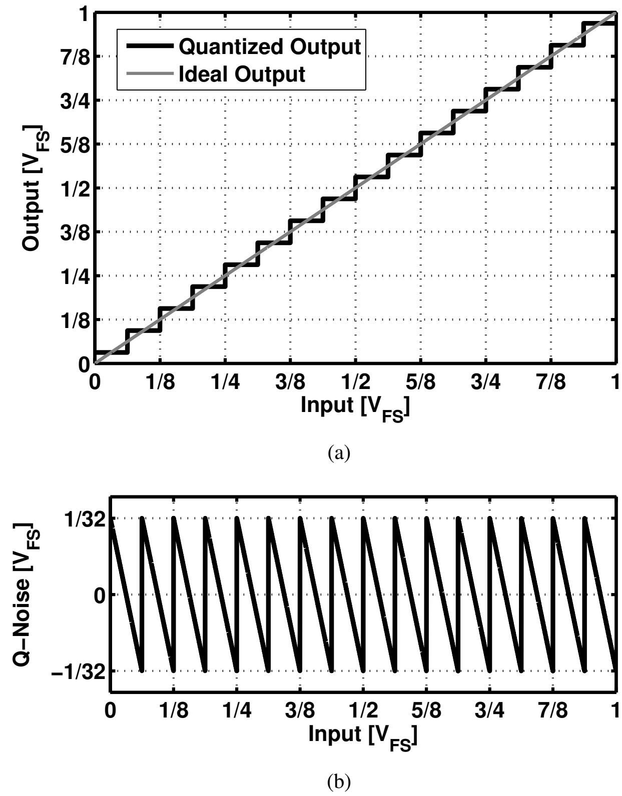

Fig. 1. a) An ideal 4-bit ideal ADC. b) The quantization noise voltage of an ideal 4-bit ADC, where quantization noise is the input subtracted from the output.

distributed between 0 and 1, the normalized range of the theoretical ideal ADC in Fig. 1(a). This signal can be described as a random variable with a uniform probability density function (PDF). The variance of a random variable is equivalent to its mean-square power, and since the variance of a uniform PDF is found to be

$$\mathrm{Var}(PDF_{uniform}) = \frac{\Delta^2}{12}, \tag{2}$$

where Δ is the range of the PDF. In the case of the signal, Δ=1 thus signal power is

$$S_{uniform} = \frac{1}{12}. \tag{3}$$

To calculate noise power, it must first be observed from Fig. 1(b) that quantization noise is uniformly distributed

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between ±1/2 LSB where

$$LSB = \Delta = \frac{1}{n+1}, \tag{4}$$

and n is number of comparators. Therefore,

$$N_{uniform} = \frac{(\frac{1}{n+1})^2}{12}. \tag{5}$$

Now that signal and noise power (mean-square) are known, SNR in terms of voltage can be found by taking the ratio of the rms values of each, i.e.

$$\begin{aligned} SNR_{uniform} &= \frac{\sqrt{S_{uniform}}}{\sqrt{N_{uniform}}} \\ SNR_{uniform} &= n+1, \end{aligned} \tag{6}$$

or in decibels,

$$SNR_{uniform,dB} = 20\log(SNR_{uniform}). \tag{7}$$

SNR is related to ENOB by the relationship

$$2^{ENOB} = SNR_{uniform}, \tag{8}$$

and since SNR can be attained in units of dB, the result is simply,

$$ENOB = \frac{SNR_{uniform,dB}}{20\log(2)} = \frac{SNR_{uniform,dB}}{6.02} \tag{9}$$

From (6) and (8) the resulting number of comparators n required for a given ENOB is

$$n = 2^{ENOB} - 1, \tag{10}$$

which is the expected result.

B. ENOB from Sine-Wave Input

For a sine-wave input such as the waveform seen in Fig. 2(a), the observed quantization noise (Fig. 2(b)) still appears to be roughly uniformly distributed and, for the most part, uncorrelated to the input signal. Therefore the noise power in the sine-wave input case is identical to (5),

$$N_{sine} = \frac{(\frac{1}{n+1})^2}{12}. \tag{11}$$

The signal, however, is no longer a uniform distribution, but a sine-wave distribution whose PDF is

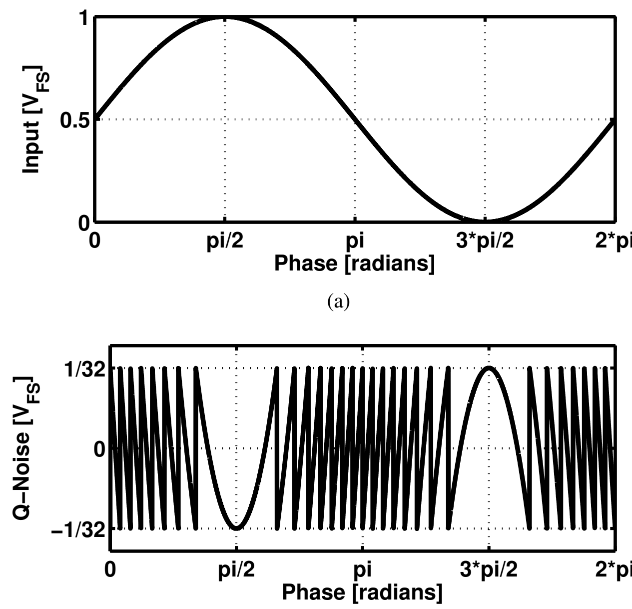
$$PDF_{sine}(v) = \frac{1}{\pi\sqrt{\frac{1}{4} - (v-\frac{1}{2})^2}} \tag{12}$$

over the full-scale range from 0 to 1. The variance of (12) is

$$\mathrm{Var}(PDF_{sine}) = \frac{1}{8}, \tag{13}$$

and thus the signal power is

$$S_{sine} = \frac{1}{8}. \tag{14}$$



(a)

(b)

Fig. 2. a) A full-scale sine-wave input. b) The associated quantization noise voltage for an ideal 4-bit ADC.

Calculating SNR yields

$$\begin{aligned} SNR_{sine} &= \frac{\sqrt{S_{sine}}}{\sqrt{N_{sine}}} \\ SNR_{sine} &= \sqrt{\frac{3}{2}}(n+1). \end{aligned} \tag{15}$$

The result from (15) is not the same as (6). Since SNR should be in terms of a random, uniformly distributed input, a conversion factor must be applied,

$$SNR_{uniform} = \sqrt{\frac{2}{3}}SNR_{sine}, \tag{16}$$

or in decibels,

$$\begin{aligned} SNR_{uniform,dB} &= SNR_{sine,dB} + 20\log(\sqrt{\frac{2}{3}}) \\ SNR_{uniform,dB} &= SNR_{sine,dB} - 1.76. \end{aligned} \tag{17}$$

It is from (17) and (9) that we derive (1), repeated here,

$$ENOB = \frac{SNR_{sine,dB} - 1.76}{6.02}. \tag{18}$$

III. UNIFORM DISTRIBUTION STOCHASTIC ADC

Now consider an ADC whose comparator thresholds are randomly and uniformly distributed along full-scale from 0 to 1. This ADC scenario has been demonstrated to exist in [1], where two Gaussian distributions are used to generate a virtual uniform distribution. A possible example of a 3-comparator ADC of this type is shown in Fig. 3. Calling this a 2-bit ADC would be a misnomer, since quantization noise is actually much higher than an ideal 2-bit ADC, as will be

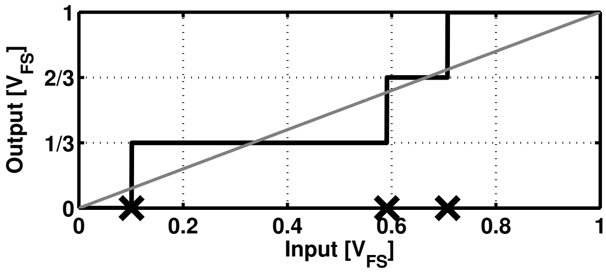


Fig. 3. A 3-comparator ADC with random comparator placement. The marks along the x-axis represent the comparator thresholds

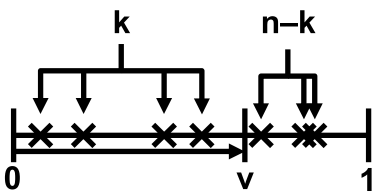


Fig. 4. Normalized transfer function of a basic stochastic flash ADC with uniformly distributed comparator offsets for three cases where n is the number of comparators.

demonstrated in this section. In order to bound quantization noise to zero at the extremes of the input, let

$$LSB = \frac{1}{n}. \tag{19}$$

Fig. 4 is a visualization of how to describe the random comparator placement as a random variable. If it is known that there are n comparator thresholds within the range 0 to 1, and the number of thresholds between 0 and an arbitrary point v is called k, then the remaining thresholds $(n-k)$ must exist between v and 1. Since these comparator thresholds are uniformly distributed, the random variable k is a binomial distribution [2] with a probability-mass-function (PMF) of

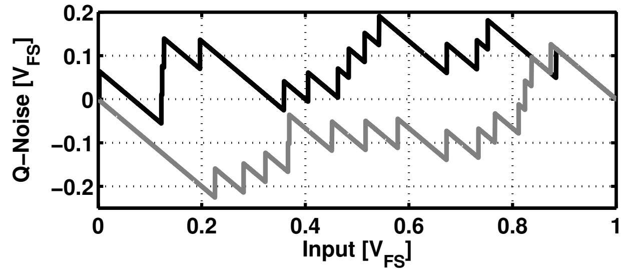
$$\mathrm{PMF}_k(n,v) = \binom{n}{k}(v)^k(1-v)^{n-k}. \tag{20}$$

For any given value of input v, the output is merely k/n since $1LSB$ was defined as $1/n$. The mean of the output k/n as a function of n is calculated to be

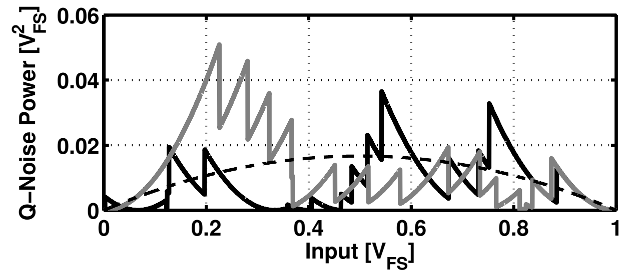
$$\begin{aligned} \mathrm{Mean}(\frac{k}{n}(v)) &= \sum_{k=0}^{n}\left(\frac{k}{n}\right)\binom{n}{k}(v)^k(1-v)^{n-k} \\ \mathrm{Mean}(\frac{k}{n}(v)) &= v. \end{aligned} \tag{21}$$

This result means that, for example, in the range 0 to 30% of full-scale there will on average be 30% of the total comparator thresholds. Any variation from this mean value results in quantization noise, therefore the variance of k/n is quantization noise power as a function of input, and is calculated to be

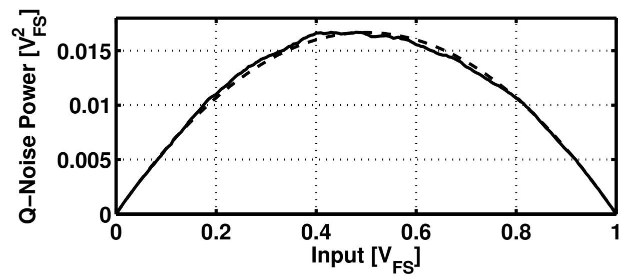
$$\begin{aligned} \mathrm{Var}(\frac{k}{n}(v)) &= \sum_{k=0}^{n}\left(\frac{k}{n}-v\right)^2\binom{n}{k}(v)^k(1-v)^{n-k} \\ \mathrm{Var}(\frac{k}{n}(v)) &= \frac{v-v^2}{n}. \end{aligned} \tag{22}$$



(a)



(b)



(c)

Fig. 5. a) Shown is the quantization noise of two random 15-comparator ADCs with uniformly distributed comparator thresholds (and $1LSB$=1/15). b) This shows the square of quantization noise of the same two random ADCs. The dashed line is a plot of (22). c) This is the square of quantization noise of 5000 random ADCs averaged together. The dashed line is a plot of (22).

A. ENOB from Uniform Input

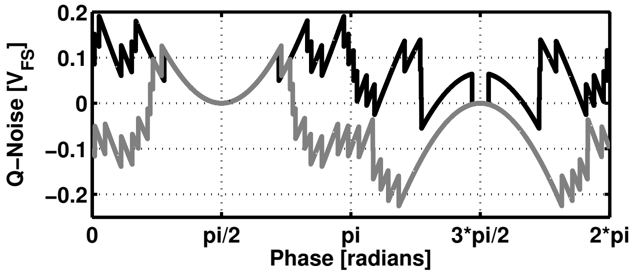
The signal power for a uniformly distributed input is the same as in the ideal case, only the quantization noise will be different, i.e

$$S_{uniform} = \frac{1}{12}. \tag{23}$$

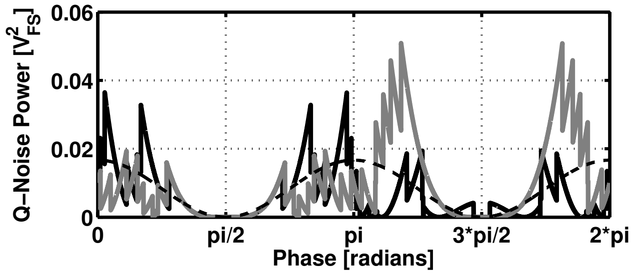
Quantization noise power as a function of input was found in (22), and integrating this function over the signal range gives total quantization noise power;

$$N_{uniform} = \int_0^1 \frac{v-v^2}{n}dv = \frac{1}{6n}. \tag{24}$$

This result can be verified numerically as in the example in Fig. 5.



(a)



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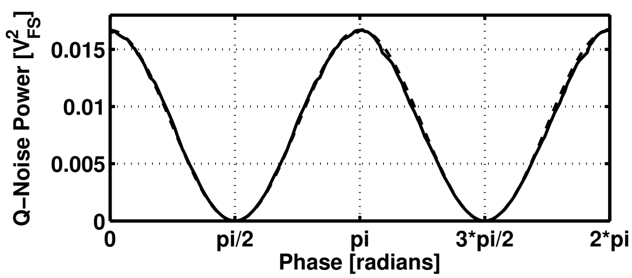

(c)

Fig. 6. b) This is the quantization noise due to the sine-wave input in Fig. 2(a) for the same two random ADCs as in Fig. 5. c) This shows the square of quantization noise of the same two random ADCs. The dashed line is a plot of (22) with $v = (1/2)\sin(x) + 1/2$ and $x = 0\ldots2\pi$. d) This is the square of quantization noise of 5000 random ADCs averaged together. The dashed line is again a plot of (22) with $v = (1/2)\sin(x) + 1/2$ and $x = 0\ldots2\pi$.

SNR is finally calculated to be

$$\begin{aligned} SNR_{uniform} &= \frac{\sqrt{S_{uniform}}}{\sqrt{N_{uniform}}} \\ SNR_{uniform} &= \sqrt{\frac{n}{2}}. \end{aligned} \tag{25}$$

B. ENOB from Sine-Wave Input

As in the case of an ideal ADC, the signal power of a sine-wave input is

$$S_{sine} = \frac{1}{8}. \tag{26}$$

The quantization noise power, however, changes due to the input. Recall that from (22) (shown graphically in Fig. 5(c)), quantization power is smaller toward the extremes of the input range, and that a sine-wave spends more of its time at these regions. Integrating (22) multiplied by the PDF of a sine-wave weights the quantization noise power by the non-uniform distribution of the input and yields

$$N_{sine} = \int_0^1 \left(\frac{v - v^2}{n}\right) \left(\frac{1}{\pi\sqrt{\frac{1}{4} - (v - \frac{1}{2})^2}}\right) dv = \frac{1}{8n}. \tag{27}$$

Another way of finding the same result is to substitute the actual sine-wave signal for v in (22) and integrate over the time period of the signal, yielding

$$\begin{aligned} N_{sine} &= \frac{1}{2\pi} \int_0^{2\pi} \frac{(\frac{1}{2}\sin(t) + \frac{1}{2}) - (\frac{1}{2}\sin(t) + \frac{1}{2})^2}{n} dt \\ N_{sine} &= \frac{1}{8n}. \end{aligned} \tag{28}$$

This result can also be verified numerically as in the example in Fig. 6.

SNR for sine-input is finally calculated to be

$$\begin{aligned} SNR_{sine} &= \frac{\sqrt{S_{sine}}}{\sqrt{N_{sine}}} \\ SNR_{sine} &= \sqrt{n}. \end{aligned} \tag{29}$$

The result from (29) is not the same as (25). As in the ideal ADC case, SNR should be in terms of a random, uniformly distributed input, thus a conversion factor must be applied,

$$SNR_{uniform} = \sqrt{2}SNR_{sine}, \tag{30}$$

or in decibels,

$$\begin{aligned} SNR_{uniform,dB} &= SNR_{sine,dB} + 20\log(\sqrt{2}) \\ SNR_{uniform,dB} &= SNR_{sine,dB} - 3.01. \end{aligned} \tag{31}$$

This result implies that for an ADC such as described here, ENOB calculation from a sine-wave input should actually be

$$ENOB = \frac{SNR_{sine} - 3.01}{6.02}. \tag{32}$$

IV. Conclusion

The equation for calculating ENOB from SNDR of a sine-wave test was derived for an ideal ADC and a uniform stochastic ADC. The fact that the two results are different should bring into question the accuracy of ENOB specifications for ADCs that do not have quantization noise (or any noise, for that matter) that is uncorrelated to the input. If noise is correlated to the input then using the conventional ENOB calculation may not be correct and a derivation such as the one is this paper is required.

Acknowledgment

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[2] H. Stark and J. W. Woods, *Probability and Random Processes with Applications to Signal Processing*, Prentice Hall, 2001. Ch. 1, pp. 40-41